The Wolf Prize for Mathematics, the third most prestigious distinction in Mathematics after the Abel Prize and the Fields Medal, has been awarded in 2019 jointly to Jean-François Le Gall from the University Paris Sud "*for his profound and elegant works on stochastic processes*", and to Gregory Lawler from Chicago University. This is the brightest award of an already impressive list of academic recognitions that Jean-François Le Gall has received: he gave a plenary lecture at the International Congress of Mathematicians in 2014, is a member of the French Academy of Sciences since 2013, was awarded the Silver Medal of CNRS, the Fermat Prize, the Loeve Prize and the Sophie Germain Prize, ... We seize this opportunity and review succinctly in reverse chronological order some of his main achievements in the field of Probability Theory.

- **Random planar maps and the Brownian map.** The Central Limit Theorem states that after proper centering and rescaling, the sum of n independent copies of a random variable with finite variance converges in law as $n \to \infty$ to a standard Gaussian distribution. Donsker's invariance principle provides a functional version of the latter; it asserts that the Wiener measure, that is the distribution of the Brownian motion, arises as the scaling limit of any random walk with finite variance. Even though there exists no Haar measure on the infinite dimensional space $\mathcal{C}(\mathbf{R}^d)$ of continuous paths $\omega : [0, \infty) \to \mathbf{R}^d$, Donsker's invariance principle suggests to view the Wiener measure as the natural "uniform" probability measure on $\mathcal{C}(\mathbf{R}^d)$. In turn, the Brownian map is a random metric space that can be thought of, loosely speaking, as a uniform random surface homeomorphic to the 2-dimensional sphere $\mathbb{S}^2$. There is a variety of reasons, notably from quantum gravity in Theoretical Physics, why one should be interested in such objects, and Jean-François Le Gall's contributions in this area are groundbreaking.

  Informally, he established in a series of works the counterpart of Donsker's invariance principle for surfaces instead of paths. More specifically, he solved the crucial problem raised by Oded Schramm of establishing the convergence in distribution of several discrete models of random planar geometries, such as uniform random quadrangulations[1] with n faces of $\mathbb{S}^2$, to the Brownian map (the problem of the uniqueness of the Brownian map was solved independently and at the same time by Grégory Miermont).

  Jean-François Le Gall has established a number of deep features of this new object. He proved that its Hausdorff dimension is 4 almost surely, that it enjoys a remarkable invariance property after uniform random re-rooting, described precisely its geodesics, etc. The Brownian map and its relatives form one of the most important and active fields of research on stochastic phenomena now-a-day, and their connexions to some other fundamental objects including Conformal Loop Ensembles and Gaussian Free Field, confirm their central role in Probability Theory.

[1] A quadrangulation of the sphere is a proper embedding of a finite connected graph in the sphere, without edge-crossings, and such that all faces are bounded by exactly 4 edges.

- **Branching processes.** The branching property lies at the core of many contributions of Jean-François Le Gall. Informally, it refers to a basic assumption that is very frequently made in random populations models, namely that different individuals evolve independently one from the other. Although this notion is easy to formalize when reproduction events do not accumulate, because then there is no difficulty for defining the genealogical structure in terms of a discrete tree, dealing with the continuous setting is often much more complex. Probably the most remarkable contribution of Jean-François Le Gall in this area is the introduction of the Brownian snake, a process with values in a space of trajectories, which is a tool of fundamental importance for solving a variety of problems.

  In short, at each time $t \geq 0$, the Brownian snake W_t is a Brownian trajectory with random lifetime $\zeta(t)$, where the process $(\zeta(t) : t \geq 0)$ is a reflected linear Brownian motion. Informally, for each infinitesimal time increment $\mathrm{d}t$, if $\mathrm{d}\zeta(t) = \zeta(t + \mathrm{d}t) - \zeta(t)$ is negative, then $W_{t+\mathrm{d}t}$ is given by the restriction of W_t to the time interval $[0, \zeta(t + \mathrm{d}t)]$, whereas if $\mathrm{d}\zeta(t)$ is positive, then $W_{t+\mathrm{d}t}$ is obtained by extending the path W_t from its right-extremity and patching an independent Brownian path of duration $\mathrm{d}\zeta(t)$.

  The Brownian snake yields notably a most useful construction of a class of measure-valued Markov processes called super-processes, from which a number of features of the latter can be derived. In turn, as it was initially stressed by E.B. Dynkin, super-processes have deep connexions to certain non-linear PDE's (typically $\Delta u = u^2$), and the Brownian snake can be used to establish fine properties about the singularities of the solutions, their asymptotic behaviors close to the boundary in situations when the solution explodes, etc. In a somewhat different direction, together with Yves Le Jan, Jean-François Le Gall constructed continuous state branching processes with a general (i.e. non-binary) branching mechanism from Lévy processes (i.e. processes with independent and stationary increments) without negative jumps. This enables to extend the construction of the Brownian snake to Lévy snakes, and to generalize many results which have been established previously for the binary branching mechanism to general ones. Furthermore, the genealogy of a Lévy snake is described by a continuous random tree, called a Lévy tree, which is a close relative to the celebrated (Brownian) Continuum Random Tree of Aldous, and has appeared since in many limit theorems for random structures. Last but not least, it should be stressed that the Brownian snake plays also a key role in the construction and the study of the Brownian map.

- **Intersection of planar Brownian paths.** Some of the earliest contributions of Jean-François Le Gall delt with multiple points of the planar Brownian motion or of the simple random walk. One says that a path $\omega \in \mathcal{C}(\mathbf{R}^d)$ possesses a point of multiplicity (at least) $k \geq 2$ if there exist times $0 \leq t_1 < \ldots < t_k$ such that $\omega(t_1) = \ldots = \omega(t_k)$. Since the works of Dvoretzky, Erdös and Kakutani in the 50's, it is well-known that the

Brownian path in $\mathbf{R}^d$ has almost surely no double points when $d \geq 4$, possesses almost surely double points but not triple points when $d = 3$, and points of arbitrary (even infinite) multiplicity in dimension $d = 2$. By building upon the concept of intersection local time, which had just been introduced by Jay Rosen, Jean-François Le Gall obtained a number of remarkably fine results in this setting. In particular, he has been able to determine the exact Hausdorff function for the set of points with a given multiplicity. He also enlightened the role of the self intersection local times and its renormalization in the asymptotic study of the area of the so-called Wiener Sausage (the trace left by some compact set translated along a Brownian trajectory), considerably refining an earlier result due to Kesten, Spitzer and Whitman in the 60's.

Jean-François Le Gall is one of the brightest and most influential probabilists of his generation. This brief description of some of his major contributions is of course far from being complete, and his mathematical works covers many more interesting topics.