An interview of Gabriel Peyré, a CNRS senior researcher at the Département Mathématiques et ses Applications at Ecole Normale Supérieure (CNRS / ENS Paris). Gabriel Peyré is an invited speaker at ECM 2020.

What's your research domain?

I am an applied mathematician. I have two main domains of application: imaging, including for example medical, astrophysical and biological imaging, and machine learning, which consists in programming machines into performing tasks. The tools I mainly use deal with optimal transport. Optimal transport dates back to the 18th century, when the French mathematician and politician Gaspard Monge addressed the military problem consisting in finding the optimal way of shifting excavated earth from one place to build embankments elsewhere. He was able to formulate the problem but not to solve it. Leonid Kantorovitch solved the problem in the 1940s in an economical context and was awarded the Nobel Prize in Economics in the 1970s for his discoveries in optimal transport. For the past four or five years, researchers have been using this method in the domains of imaging and machine learning.

What's your contribution to those domains as a mathematician?

Imaging and machine learning require computer scientists, engineers and mathematicians. As a mathematician, I shed a theoretical and methodological eye at each level, firstly by modelling the problems; secondly by developing mathematical and numerical methods to solve them; thirdly, by developing algorithms - now, that is the computer science part but yielding an efficient algorithm requires to understand the structure of the problems, their geometry, i.e. the relationships and the hierarchy between the objects, and that's where the theoretical approach of mathematics is essential. Lastly, we applied mathematicians sometimes collaborate with scientists of other disciplines who make the experiments and manipulate the data for example in medical imaging, neurosciences, genomics or biological imaging. We then apply our methods to real cases and try to understand what is at stake in those problems to take it into account and adapt our methods accordingly.

What subjects do you tackle these days?

For some four or five years, I have been dealing with optimal transport to better understand and apply neural network methods to the domains of imaging and machine learning. The question lying at the core of human learning is this one: how does the brain work? And its machine learning replica is: should machines be programmed as the human brains works or otherwise? The gap between human and machine learning is deep and one does not know how to handle it yet. Since 2012 the neural network method suddenly reappeared. It proved to be the method that worked in machine learning and had an impact on all scientific disciplines. What is at stake on a mathematical point of view is this: those methods work without our knowing why, and understanding them requires to shed a mathematical, methodological approach of their performance. And this can be done with optimal transport theory.

What is at stake in such researches?

The field of learning is a scientific field that is booming, in particular because there are major industrial issues for all companies, technological and other, which have to deal with large sets of data, images, videos and texts. Social networks, for example, have huge masses of data in text and social relations. To understand them, you need huge networks that allow you to automate all tasks, classify images, translate texts, recognize images, translate video subtitles, generate computer-generated images, create virtual animations... Learning also affects the automotive industry with the issue of the autonomous car. There are also impacts in more theoretical science in the sense that neural networks will be used to understand theories of physics, to analyze data in biology... But these are very complex methods, which we do not really know how to master. And it raises the question of what a neural network really is: how can we know exactly how the network works and how to use it in a way that we can have some assurance what the network does?

How does a neural network work?

The neural network method is an algorithm. An algorithm is a very general concept: it's something you can put on a computer and which will perform a task. Classical algorithms are programmed by programmers. They depend on the efficiency of their programmers. Learning algorithms, of which neural networks are the best known, are not so complicated in themselves, rather, they adapt to the data they are given as examples. A learning algorithm is being fed with a large number of examples such as ‘this is a car’, ‘this is a dog’ etc. millions of times, and it automatically adapts itself, changing its internal components, to solve the task as efficiently as possible. The characteristic of learning algorithms is that they need a lot of data and a lot of computing time to adapt themselves automatically. Now, the novelty about all this is, that the world is going to be dominated or regulated by algorithms that learn from data and are no longer actually programmed. We still need to continue training programmers, but we also need to train people who are able to understand how the algorithms use the data to program themselves.

What is the role of mathematics in the development of algorithms?

The development of an algorithm requires mathematics at all levels, not only to understand and formalize the problem that the algorithm will have to solve, but also to help develop an efficient algorithm. Say, for example, that there are several possible algorithms to solve a task and the goal would be to pick an algorithm that either goes as fast as possible or gives the best possible solution: to understand the structure of the problem, to analyze it, to break it down into sub-problems and understand how the different problems depend on each other, i.e. to understand the geometry of the question, you’ll need mathematics. It's hard to think of developing very efficient algorithms if you don't do mathematics at that level too. And once you have developed an algorithm that you think is efficient, you must also be able to yield a theoretical certification of it, that is, to prove it mathematically. And you need to understand theoretically the cases where the algorithm does not work or does not work well, because it is crucial to point this out in order to be able to improve the algorithm.

How did you become a mathematician?

I have had a rather standard career. As a child I was good at math but for sure I did not do math by default. What attracted me in mathematics is the creative side, just like when you write dissertations for the literature course. I enjoyed seizing the problems as a game, analyzing them and solving them. It was exciting because there are questions you don't exactly know how to solve. I believe I always knew that mathematics was what I wanted to do.

Would you say there is a distinction to be made between mathematics and applied mathematics?

It doesn't seem efficient to me to draw a mathematical landscape where applied mathematics would be distinct from fundamental mathematics. You may need the distinction to classify people but that's all. Applied mathematics are mathematics. It does not consist in applying existing mathematical concepts to real problems – that's what engineers or, say, physicists, do. An applied mathematician creates new mathematics, new theories, he studies them, develops them and also grabs the opportunity to confront them to real problems, and to do that in interaction. On a daily basis, it's very relative: you sometimes have the posture of a very theoretical mathematician, explaining mathematics which seems very theoretical to another person, and sometimes you discuss with people who do very abstract mathematics and you're also very comfortable discussing very abstract mathematics with them. Our role as an applied mathematician varies according to our collaborations and interactions, and to the problems we solve and the questions we study.

Do you often work in collaboration?

I conduct most of my research collaboratively. I've never picked and solved a problem from beginning to end on my own. There are times when I'm working on my own, but at some point, I always interact with collaborators. These are my students, my colleagues, researchers from other fields, or even people working in industry although I don't actually have proper industrial collaborations.

The scientific environment is important: attending conferences, talking over coffee... that's how projects start. You can't decide to collaborate on a subject with so-and-so because he is a physicist, for example. Collaborations start on a casual, ordinary opportunity where you rough out a problem with someone and if there is a convergence point of common interest, it may develop into a collaboration. It's important to have a lot of interaction, so that it can lead to collaborations from time to time. And what's also interesting about collaborations is that you can connect people to each other, advising someone to go contact some other researcher on such aspect because that person is a specialist.

I have a lot of different interlocutors because I enjoy learning new, different, both theoretical and applicative aspects. And also, I enjoy working in parallel on several different subjects. Now, one should not disperse… It's necessary to be sure that you will be able to make progress on all the subjects you're engaging upon. From that point of view, collaborating with students is supportive. They call our attention to questions and problems every day!

What's the use of mathematics?

Firstly, mathematics is necessary in all areas of science. It is important to keep that in mind. Now, one often hears that mathematics is not needed in machine learning, since “it works”. But my opinion is that mathematics provides the theoretical background to go deeper into problems, to model them and to certify methods. You won’t move forward as quickly nor build a strong construction without mathematics, even in areas where it seems to work out very well without it. Mathematics is there to bring coherence, which can be quite complicated in science, and also to bring language. Someone who does not master the language of mathematics will find it difficult to formulate the problems. Mathematics is *the* language of science, but there is more to it. You don’t do mathematical demonstrations just for the sake of elegance. Mathematics gives you a theoretical hindsight on the context, which then helps you design even more complicated objects. And that is needed everywhere. In medicine, for example, or in artificial intelligence, from the moment you want to tackle complicated concepts, you are much faster if you write them concisely with a universal language. That's not what all mathematics is about, but providing a language and a way of constructing objects is the first step. And then certifying the things that we do to provide a theoretical guarantee allows us to build a solid construction that we will be able to grow further.

You have your own tweeter account. What’s the place of dissemination of mathematics in your job as a mathematician?

Transmitting knowledge, disseminating science, is very important to me because it is so important that young people choose to do science. I’m invited in junior high schools and high schools, I give talks to the general public, I organize conferences for the general public, for example the thematic trimester on the mathematics of imaging at the Institut Henri Poincaré in Paris last year. Interacting with young people is very enriching. Whether they have a mathematical background or not, they always ask very efficient questions and some of those might prove to be quite deep. Either one knows the answer and it’s a question of rephrasing things or considering them from another viewpoint. Or one cannot answer and it may open up a problem which may reveal itself interesting. I also participate in activities with the association TRACES. I write articles in *Images des mathématiques*, in *Horizons* and in the journal of the APMEP association which aims at high school teachers and that is important as they are the ones who are in contact with the young people and it is part of their continuing education as teachers.

My posts on tweeter address the scientific community rather than the young people – anyone who has a sufficient knowledge to deal with equations. I post an article related to mathematics and a note on an applied mathematics concept every two days. ‘Oldies but goldies’ are over 20 years, if not 300 years old scientific articles, if not forgotten articles, by Gaspard Monge and others. As for the conceptual posts, they may be related to problems

I’m working on, and writing about them helps me progress. They may be things I’ve been keeping in my drawers for years, tests I’d been doing alongside my work… so that my tweeter account is a great opportunity to make the best of everything one does alongside one’s research. And for each one, I create an original .gif animation.

What message would you like to get across to the younger generations?

I would like to tell the younger generations that there is one booming mathematical domain with scientific, industrial and societal challenges, which is not about to disappear: machine learning or artificial intelligence. That field will provide work over the next twenty years, but not only that: it is a field to invest in, with highly interesting and exciting mathematical questions, and innovative solutions to make out. Whereas we know some of the mathematics we need to use to understand things in a field like physics, in learning and in neuroscience for everything related to human learning, we do not yet know what mathematics is involved. We have to invent it. Machine learning is a new frontier. France is very competent on those issues because its mathematical school is very strong. Applied mathematics is very developed in France. And we need mathematicians. Both male and female! The fact that there are so few women involved in science is very damaging to science.

What does it mean to you to be an invited speaker at the ECM?

It's the first time I've been invited to a conference like that. It is of course an honor, and a recognition too. If there's one message I might wish to get across, it would be to invite young people - those invitations always come a little too late in a career. It would be an encouragement to young researchers to be invited and it would put them to the fore.