

An interview of Eva Miranda, full Professor at UPC, chercheuse affiliée at the Observatoire de Paris, and an invited speaker at the 8th European Congress of Mathematicians scheduled for July 2020 and postponed to June 2021.

Questions:

1. What is your research domain?
2. How would you describe your research domain to a non-mathematician?
3. Why should it take so much effort to “get out of your (mathematical) box”?
4. Can you give us an example of this combining of mathematical techniques out of your own experience?
5. What would you say is the benefit of this?
6. How did you come to hold a position as a chercheuse affiliée at the Observatoire de Paris?
7. What led you to do mathematics?
8. How did you experience being a woman involved in a mathematician career?
9. Would you say young women who don't trust themselves to do mathematics or is it an opinion which is imposed on them?
10. Are there mathematicians, male or female, that inspire you?
11. You wrote an article on Maryam Mirzakhani in *El País* in 2018. Would you like to tell us more about her?
12. What about dissemination and talking to society?

1. What is your research domain?

I am a mathematician particularly interested in understanding connections between different areas such as Geometry, Dynamical Systems, Mathematical Physics and, more recently, Fluid Dynamics.

My research is at the crossroads of Differential Geometry, Mathematical Physics and Dynamical Systems. My area of expertise is Symplectic and Poisson Geometry. I work with objects appearing on the interface of Geometry and Physics such as integrable systems and group actions acquainting for symmetries of the systems. My research deals with geometrical and dynamical aspects of the singularities arising in Symplectic and Poisson manifolds as well as with mathematical models for their quantization.

2. How would you describe your research domain to a non-mathematician?

A brief general answer would be, my research domain is geometry and topology. Let's describe it as the mathematics that studies the forms. More generally I'm working in differential geometry. That is, if you had to put me in a classifying box that would be my box. But I don't like to be inside a box. In fact, what I have been doing quite intensively lately, is to get connections between my work in differential geometry and dynamical systems and try to yield applications. For instance, in the past years I have been trying to find applications of singularities appearing in Symplectic and Contact geometry to actual problems in celestial mechanics often related to collision. Lately, for a year or so, I have started working on some

connections with fluid dynamics, which is a totally new subject for me, in collaboration with my colleague Daniel Peralta Salas. Inside differential geometry I'm working more specifically on symplectic and Poisson geometry, which is even a smaller box inside the box. Poisson Geometry started with a bracket of functions coined after Denis Siméon Poisson and is used to describe laws of physics. All these mathematical worlds are mainly motivated by problems in physics.

So, I am working on this interaction: both inside mathematics with dynamical systems and fluid mechanics, and outside mathematics with problems in mechanical systems because I like creating links between boxes. It requires energy to get out of your own box but it's worth it. It's very refreshing. You never get bored.

Of course it's very pleasant to be working in one's own box (confined in your own mathematical box), but it is necessary to leave the comfort zone and check what happens out there: in real life, in physical or mechanical problems. In doing so we can indeed find new applications. Sometimes the physicists like the models that mathematicians may suggest (for instance in quantization), sometimes they don't. In any case, although there is the cliché that mathematicians and physicists don't get along well together, they actually cooperate quite well. I think we can learn a lot from each other if we try hard.

3. Why should it take so much effort to “get out of your (mathematical) box”?

In science, one tends to be specialized in a subtopic. You get a PhD in a subtopic and you become the “queen” or the “king” of that subtopic. It is more comfortable to stay there and grow your bibliography in this box and you can become the master of this tiny little box. But I think as a scientist we have the obligation to get out of the box, look around, interact and build by learning. It takes energy in two ways. First, it is the time and energy it takes to learn a new subject from scratch (but now you learn in a more evolved way that you did when you were a PhD student. Your approach to learning is more structured. You can approach the new problems with your own expertise and perspective of science). Moreover, while you're spending that time and energy on a new subject, your colleagues who are applying to the same academic positions as you are, are sticking to their initial subject and publishing papers, so that there's a career issue to that choice too. The second factor that requires extra energy for the “out-of-the-box attitude” is the social energy, approaching different groups of people you need to interact with, go to different conferences, meet different groups of people and understand the micro social rules inside those groups. It is something I experienced when I was a postdoc in France, in Toulouse, back in 2004-2005. I had done a PhD in integrable systems and Lagrangian foliations and then I moved into Poisson Geometry. It was a lot of work for me but it paid off many years later. I got expert perspective comparing different subjects.

Some may think it is comfortable to stay in your little box but I don't think it is very exciting. I think science is about getting out of the box, maybe trying to address somebody else's question who is thinking at a particular problem from her or his perspective while you arrive with your own perspective and you can bring new light to the question. Maybe you make errors but you learn from them. Maybe this person will not be happy about it because he or she wanted to solve that question with analytical tools, here you come, applying your

topological tools, and you solve the problem... People may or may not be happy about it, but I think is very rewarding for the community. Big steps forward in science are done by combining techniques. That is what happened to the resolution of Poincaré conjecture (now theorem) that a 3-manifold which is compact and simply connected is homeomorphic to the 3-sphere. Simply connected manifolds are easy to describe: if you put a loop on them, you can squeeze it up to a point (like when you lace your shoes). For Poincaré's conjecture everyone was thinking in topological terms and then Perelman arrived with a solution that used geometric analysis. It was beautiful!

Poincaré himself was an example of polymath. Polymaths is what science needs now!

4. Can you give us an example of this combining of mathematical techniques out of your own experience?

Yes. I was working with Daniel Peralta-Salas and my PhD student Robert Cardona on the applications of the geometrical tools we were using in singular symplectic manifolds to problems in fluid dynamics. Peralta-Salas had been working on Fluid dynamics for a long time and, precisely we were combining his expertise with these forms on manifolds with boundary that explode when they get to the boundary and we wrote a paper about that [<https://royalsocietypublishing.org/doi/10.1098/rsta.2019.0034>]. I remember going back from Madrid to Barcelona one day sitting in the train, checking out twitter because nowadays you do find interesting information on twitter. I came across a question of Terence Tao and I thought "Let us try to solve this with our new techniques, let's look at the question". So we started to apply our techniques to this problem and invited Fran Presas to join the team as we needed to combine our techniques with the h-principle in contact geometry (one of Presas' expertises). So there we were the four of us applying fine techniques in contact topology to that question of Terence Tao. It has been an intense but exciting period!

Our problem concerned the Euler equations, which are close cousins of Navier-Stokes equations. The Navier-Stokes equations are used to model real life problems such as ocean currents or weather patterns. The Euler flows correspond to the case in which the viscosity of the fluid is assumed to be zero.

We were addressing the question concerning universality of interest for Terence Tao in connection to a novel approach he has been working on in relation to the Navier-Stokes millennium problem [<https://www.quantamagazine.org/terence-tao-proposes-fluid-new-path-in-navier-stokes-problem-20140224/>].

For this approach, it is useful to take as initial data for the Navier-Stokes equations some initial solutions of which we have some control (Turing complete). Tao introduced the concept of universality to analyze the Turing completeness of the Euler equations probably in view of this novel approach.

In our paper we prove that there exists an Eulerisable flow on a sphere of dimension 17 which is Turing complete (grosso modo, this means that the halting of any Turing machine with a given input is equivalent to a certain trajectory of the flow). This can be a bit deceiving for practical purposes as the dimension is not the "right one" (which would be dimension 3 for the analyst) but it is a first step in that direction.

How we got there is very interesting. Let me explain to you: you have a kind of dictionary connecting a specific type of flows (called Beltrami flows) corresponding to stationary

solutions of Euler flows with the realm of contact structures. This correspondence allows to get information about the Euler flows using techniques from Contact Geometry. For instance in our paper we prove that, but if you look “far away” by increasing the dimension of your manifold you can realize any dynamics as Euler flows via Beltrami (“universality”) and this is exciting. We do this by answering a naive question in Geometry: can any non-singular vector field be seen as a Reeb vector field associated to a contact structure? Of course, the direct answer is NO. However the answer becomes affirmative if the dimension of the ambient manifold can be increased. You can “embed” or plunge this vector field as a Reeb vector field of some contact structure by letting the magic of the h-principle act. Then you look back in the dictionary and this gives you the universality of Euler flows. It was like following the thread out of Ariadne’s labyrinth and we magically got out of it.

Now what is the connection to the initial problem of Navier-Stokes? The answer is Turing completeness. The solution of the Euler equations that encodes a universal Turing machine provided by our construction is stationary. We do not know if it gives rise to a global-time solution when considered as the initial condition for the Navier-Stokes equations on the 17-dimensional sphere. We believe interesting phenomena can happen close to this stationary solution but we still need to understand this.

We posted our article on ArXiv last November, and this is what I wanted to speak about in my talk at ECM. But now that the ECM has been postponed, I have a one-year bonus time to understand new aspects of the problem and talk to analysts! We have been in touch with Terence Tao about this via email and I gave a talk about this in the analysis group in Zurich. I am looking forward to the reaction of the analysts to our results, as the techniques we use are very different from theirs (they are from “a different box”) and I would like to know their point of view.

5. What would you say is the benefit of this?

There is, of course, an intellectual benefit to it but also a practical one on the long-run. Science advances this way. The current confinement gives me the perfect metaphor: here we are now, confined, hoping that science advances and experts come out with a vaccine for the virus. However, you can only have a tree if you planted a seed and provided water regularly. In other words, we would not have such good teams if we had not invested in science. It is so important to invest in pure and basic science. A priori, you never know how far you will get to but surely in the long run you will find important applications.

6. How did you come to hold a position as a chercheuse affiliée at the Observatoire de Paris?

Indeed, it is a very nice story. I have had close contact with the Observatoire a long time ago as I was participating in one ANR on Integrable Systems and entered in contact with Alain Albouy and Alain Chenciner. This was during my postdoctoral period in France but there has been a flashback recently. Since 2010 I had been working on a theoretical subject with Victor Guillemin, Professor at MIT, and other collaborators on symplectic structures with singularities. Symplectic structures govern our day-to-day as they determine Hamilton’s equations (which are the equations of the motion of a particle associated to the total energy of the system H). Sometimes in models “with boundary”, the structures stop to be regular and

explode along the boundary. They have a funny behavior. We realized at the beginning that this was a model for symplectic manifolds with boundary, which is a mathematical object. We were working on that intensively together with my friend and collaborator Jonathan Weitsman at Northeastern. The topic of b-symplectic (or log-symplectic) geometry became sort of trendy in the Poisson community. This was already very rewarding from an abstract point of view.

And then one day I was giving a talk in the Barcelona Mathematical Days in 2014. It is one of these workshops where you speak in front of people who don't belong to the same box as you. Amadeu Delshams and Carles Simó were in the room. Amadeu came to me after the talk and said: "Look, I have an example to these models. Do you know the three-body problem?" He showed me some computations and we realized that these models we had been working on with Victor from a theoretical point of view were models indeed of the three-body problem and analyzed collision in a detailed way which could not be done before. Ever since, we have been close collaborators with Amadeu and together we founded a Lab in Barcelona (the Lab of Geometry and Dynamical systems) as an energetic meeting point of Geometers and Dynamicists. I must say it is very successful and it is growing fast!

The three-body problem is theoretical. It describes the movement of three bodies that are attracted to one another at the same time. It models the movement of the planets in space. And there is an associated problem with some restrictions to the initial problem, where for example these bodies move in a plane (restricted three body problem). Now, if you think of these bodies: a planet has a certain mass, so it attracts other bodies, and it is also attracted by the other bodies. As that problem cannot be solved explicitly (non-integrable) anything you can say about its trajectories (the movement of the particles) in this three-body problem is interesting. The restricted versions model the movement of a satellite so the models are relevant to control the movement of this satellite. This research has of course lots of applications in real life.

That is where my connections with the Observatoire de Paris started. It was back in 2014 and though there are people in Barcelona working on the subject, the main center was in Paris. In 2017, I got a chaire d'excellence from the Fondation Sciences Mathématiques de Paris to study these problems. I interacted quite a lot with Alain Chenciner, Alain Albouy and Jacques Féjoz and with Qun Wang. That's where connections started. With Jacques Féjoz we organized a working group on related problems. The Chaire d'Excellence allowed me to stay in this magic place for 6 months and after that I became chercheuse associée with the Observatoire. With one of my students, Cédric Oms, who came under my chaire to stay for several months in Paris, we recently solved one of the questions we asked ourselves back then with the combination of techniques in singular symplectic geometry and Celestial Mechanics. I plan to post this result on the ArXiv this week.

7. What led you to do mathematics?

When I was at high school, I enjoyed mathematics a lot because it was like my own room where I could solve a problem with its own rules, without having to check out in real life. This made me feel free. I liked that. It is what attracted me to mathematics. I was also attracted to literature. It seems that at some point I had to make a choice between the two, but they have

many things in common: for instance, the creativity. If you enjoy creativity I think mathematics is a good place to be. Even if it has rules, it requires a lot of imagination when you want to prove a theorem, because before you prove a theorem nobody has been there before. This is not like an exam when you are given all the data and you know you need to adjust to the hypothesis. Research is much more exciting because you are in a *terra incognita*, you do not have the right hypotheses, and you need to find them on the way. It is like being lost in the woods and find your own way out by following hints. While doing so, you experience that freedom that mathematics gives you. Of course mathematics has strict rules but you need to conjecture what comes next: the unknown. You need to discover it!

People often think that mathematics has to do with numbers and computing but in general, it is more about having ideas, imagination and creativity. To be able to guess what comes after a certain stage of knowledge, to go one step further, you need some qualities, and some of them are creativity and imagination. If as a high school student, you are very curious, I think mathematics can be your choice.

8. How did you experience being a woman involved in a mathematician career?

When I was a student I was super lucky, I did not realize that there was that problem with women in math. However, this problem is 100% real and cannot be neglected. There are not enough women in mathematics and this is a problem because in real life there are men and women. Science needs both. I feel bad to be the only female speaker in a workshop (and this happens often). I realized this problem only later in my career. Women are applying for academic jobs with a very strong CV but only a few of them find their place in Academia. Why? Do we judge equally men and women? Something is clearly failing.

There is a lot to be done for women in mathematics and the first one is to attract girls from high schools to studying math. When I meet girls who are scared of choosing a mathematical career because they see so few girls in the mathematical career, I tell them: "Go ahead and prove them wrong! You'll need to fight, but prove them wrong." Whatever I can do to attract girls into mathematics, it's my pleasure to do.

There is a kind of "let's do this in 3 minutes" cool competition attitude in the math Olympiads. I am not criticizing them as such, I am just saying that some women may not be attracted by careers if they think that they may imply competition as it happens for instance in those Olympiads. Some profiles are encouraged by that type of competition, but there are counterexamples. I would like to say to these young women who are hesitant about doing math: "You women are very strong - you have the strength and the capacity to do this as your colleagues, or better than your colleagues. Go ahead and prove that you can excel as a mathematician!"

In France, there is a very low proportion of female mathematicians in more senior positions. On top of that, these women are then required to sit in committees because of the parity rules but then MCF women are overcharged as they have to sit in many more committees than their male colleagues. So trying to solve a problem generates a new one. However, I think it is fundamental for us to sit in those committees and in conference committees. When I am invited to such committees, I try to accept the invitation because it is important to be there

and make sure there are enough women who are recruited or picked as speakers. It's important that we, women, are part of the decisions and that we can decide who comes next in the career – without excluding anyone of course.

9. Would you say young women who don't trust themselves to do mathematics or is it an opinion which is imposed on them?

What I have to say about this is rather controversial. It seems to me that society plays a strong role in this. The message that social and mass media seem to be conveying is not encouraging the role of women in Science but sometimes even the opposite (which reminds me of the terrible campaign "it's a girl thing" [<https://youtu.be/GMOqpxlW66E>]). Even if there are international, European and local associations and committees which are active to attract women in mathematics, family is also very important and having the support of your family is fundamental. I think starting to work on this - on family level and smaller levels -, could be a good solution. However, I would like to stress that this is a complex problem. For reasons that we do not understand it is true that the situation is worse as you go north. And the crisis has a strong impact as in many families only one of the two parents is working. That also plays a role. In Spain the problem is different. It is very hard to get women "maître de conférence" although they are really good – and why do they stop? In Central Europe, there are female-only positions which are often very criticized. This is something you cannot have in Spain as it is not legal. Maybe we need to convince ourselves that these measures are controversial, but somehow necessary at this stage. When we choose speakers for a conference, I always try to have fifty-fifty men & women and one day I realized that my colleagues asked me to justify very hard my choice of female speakers – why this woman has to be a speaker. Maybe we have to work on that too. We often ask women much more things than we do ask a man to attain a certain stage of her career or to simply speak on a workshop. I am happy to do whatever I can to solve this problem, though sometimes it gets worse. We need to be brave in the kind of decisions we take in this area.

Last September we organized a Conference on Women in Geometry and Topology. It was very successful and it even went on the newspapers (as it is strange to find these conferences in Spain). I also organized the BGSMath day for women and girls in mathematics last year at the Institute of Catalan studies. I was super happy to see how very respectful the young boys and girls were of the image of women as scientists. Maybe we adults are the ones who make things more difficult. Maybe we should let the new generations be brave and take the lead.

I think it is very hard in general to have a career in science as a researcher. It requires a lot of efforts. It means travelling a lot to attend conferences. It is very absorbing and sometimes is a bit incompatible with having a family life, not even talking of having children. It is quite stressful to be a woman in a scientific career. However, we lose a lot of input if we do not have women in science. We need to find a way out of this maze.

10. Are there mathematicians, male or female, that inspire you?

Well, you see, I could answer Henri Poincaré right away. He is number one on the list. And I could answer Emmy Noether or Sophie Germain.

Now, on the one hand, if you want to inspire a young girl who is starting in mathematics, you may tell her about Emmy Noether or Sophie Germain... but it can be quite intimidating for her: take Sophie Germain. I find her a heroine, the strongest possible woman, even signing her letters and work in math as a man – I mean, all this effort to be a mathematician! And all these women had to have a strong support from their family because they wouldn't have a salary, being women. But on the other hand, I feel that when we tell young girls "Be like Sophie Germain", "Be like Marie Curie", we are asking them to be superheroines and this is unfair. We ask the boys: "Do you like Poincaré?" – of course they like Poincaré. It is more comfortable to identify yourself with Poincaré than with Sophie Germain. Do not take me wrong. I do like Poincaré, he is also a hero for me but of course, Poincaré had it much easier to be Poincaré probably than Sophie Germain to be Sophie Germain. Everybody was believing in what he was doing. But these women at that time, nobody would believe what they were doing. What I am saying is that if we want to inspire new generations, we should give them other models like, say, Maria Esteban, Nalini Anantharaman, Michèle Vergne, Yvette Kosmann-Schwarzbach, Maryam Mirzakhani... There is a danger in selling this idea of Marie Curie as a model because there a danger you have to believe you have to be a superhero or a superheroine to be a scientist. You need to work to be a scientist, but you do not need to be a superhero or a superheroine. You need to give young girls real models, mathematicians who are alive, close to them, who have family or chose not to have family or could not have family, but they are human beings and young girls do not see them as superheroines.

So, as I said, I take a strong inspiration in Henri Poincaré, but I also take a strong inspiration from Michèle Audin. She is now retired but still active as a writer. Back in 2000 I was attending a summer school where she was lecturing at CIRM on Hamiltonian dynamics and there I was at the beginning of my PhD thinking "It's wonderful, look, I want to be like Michèle Audin!". I still use her books for my Master course in Symplectic Geometry.

11. You wrote an article on Maryam Mirzakhani in *El País* in 2018. Would you like to tell us more about her?

El País contacted me to write an article about her when she passed away because she was also working in geometry and dynamics, although not exactly the same mathematics. It was quite demanding as it is tricky to write a paper on a mathematician for the general public.

My goal is, to organize a mathematical year dedicated to the mathematics of Maryam Mirzakhani in our school with special activities. For the time being we have had an exhibit on Maryam Mirzakhani, and our school of mathematics organized a special day where the professors and the students were explaining the pictures of the exhibit "Remember Maryam Mirzakhani". Alex Eskin, a close collaborator of Maryam Mirzakhani, honored her memory in the ceremony of the Breakthrough prize for mathematics this year. Maryam Mirzakhani died very young and already leaves an important heritage. She came from Iran, she was the first woman in her country to participate in the Math Olympiads and she went to Princeton for her PhD. Maryam Mirzakhani has the "Yes we can" ingredient. She is only woman who won the Fields Medal. The thing that struck me, when she went to take the Fields Medal, is that she took her daughter Anahita and her husband, who is a mathematician, with her. She took her family with her and this is not very standard. Unfortunately, she died so she is no longer a living model for young people but she has all the ingredients. And she's still closer to our schoolgirls.

I am so pleased that you are publishing this interview online on her day May 12! Thanks a lot!

12. What about dissemination and talking to society?

Dissemination is one of the things it is so difficult to convince people to do. It is very important to go to those open spaces in schools and talk about what we are doing.

I was supposed to go to CosmoCaixa on March 18th and talk to a big audience of people about imaginary numbers and it was cancelled because of the epidemic. I try to engage in dissemination activities. It takes time because you're talking to people who are generally non-mathematicians and you need time to think about how you're going to present them such or such mathematical object.

It is an effort we need to do. More people should get involved. We, scientists, ask the government to give us money for our research. It is difficult to get some money and one way to convince the government is to go to the society and tell them why we are important as mathematicians. We need to make the effort of getting closer to all these people – maybe they'll never become mathematicians, maybe they will. But they are part of society, society is giving money for research, and telling the society what we're doing is a kind of obligation.

It can take the form of a talk in a bar about symmetry – as I did for Pint of science –, or writing in El País about Maryam Mirzakhani [https://elpais.com/elpais/2018/07/11/ciencia/1531326711_222889.html] or about Marina Viazovska [https://elpais.com/elpais/2018/08/14/ciencia/1534246378_288233.html] and her solution to the problem of the packing of spheres. Last year Juan Margalef was a postdoc in our lab and he engaged all my PhD students and myself in Pint of science, Women in science etc. He did a wonderful speech about women in science during the Women in Science day. His enthusiasm for dissemination is contagious and he got me into it.

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Eva Miranda in Shanghai in May 2019 (courtesy of Juan Margalef)